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ELECTROMAGNETIC DECAYS OF VECTOR MESONS IN A COVARIANT MODEL

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Abstract

A fully covariant model for describing the electromagnetic decay of Vector Mesons, both in light and in heavy sectors, is presented. The main ingredients of our approach are i) an Ansatz for the Bethe-Salpeter vertex for Vector Mesons, and ii) a Mandelstam-like formula for the electromagnetic decay constant. The free parameters of our approach are fixed through a comparison with the transverse momentum distribution obtained within a Light-Front Hamiltonian Dynamics framework with constituent quarks. Preliminary results for both the decays constants and the probability of the valence component are shown.

1 Introduction

Aim of this contribution is to present a fully covariant model for describing the electromagnetic (em) decay of Vector Mesons (VM's), both in light and heavy

sectors. To this end, a simple analytic form for the Bethe-Salpeter (BS) amplitude of VM's is adopted in order to perform without any further approximation the calculations of the decay constants. Moreover, with such an Ansatz one can easily evaluate the so-called transverse momentum distribution of a constituent inside the VM (see de Melo et al ¹⁾ for the pion case), that plays an essential role for fixing the value of the parameters appearing in our approach, and in turn for including some non perturbative inputs in our analytical Ansatz. A possible form of the BS amplitude for an interacting $q\bar{q}$ system with $J = 1$, can be written as follows

$$\Psi_\lambda(k, P) = S(k, m_1) [\epsilon_\lambda(P) \cdot V(P)] \Lambda_{VM}(k, k - P) S(k - P, m_2) \quad (1)$$

where $S(p, m)$ is the Dirac propagator of a constituent with mass m , P^μ the four-momentum of a VM with mass $P^2 = M^2$, $\epsilon_\lambda^\mu(P)$ its polarization four-vector, λ the helicity, $V^\mu(k, k - P)$ the Dirac structure of the amplitude and $\Lambda_{VM}(k, k - P)$ the momentum dependence of the BS amplitude. In particular, the adopted covariant form for the Dirac structure is the familiar one (transverse to P^μ), viz

$$V^\mu(P) = \frac{M}{M + m_1 + m_2} \left[\gamma^\mu - \frac{P^\mu \not{P}}{M^2} + i \frac{1}{M} \sigma^{\mu\nu} P_\nu \right] \quad (2)$$

that in the limit of non interacting system leads to the Melosh Rotations for a 3S_1 system ^{2, 3)}, as expected. For the present preliminary calculations, the momentum dependence has the following simple form with single poles, viz

$$\Lambda_{VM}(k, k - P) = \mathcal{N} \frac{[k^2 - m_1^2 + (P - k)^2 - m_2^2]}{\prod_{i=1,3} [k^2 - m_{R_i}^2 + i\epsilon] [(P - k)^2 - m_{R_i}^2 + i\epsilon]} \quad (3)$$

where m_{R_i} , $i = 1, 2, 3$ are the free parameters of our Ansatz (to be determined as described below), \mathcal{N} the normalization factor, that can be derived by imposing the standard normalization for the Bethe-Salpeter amplitude, in Impulse Approximation (i.e. with free propagators for the constituents). The form chosen for $\Lambda_{VM}(k, k - P)$ allows one both to implement the correct symmetry under the exchange of the quark momenta (for equal mass constituents) and to avoid any free propagation of the constituents (cf the numerator in Eq. (3)).

To determine m_{R_i} in Eq. (3), we first define the constituent transverse momentum distribution inside the VM, $n(k_\perp)$, along the same guidelines

adopted by de Melo et al.¹⁾ for the pion, within a Light-Front Hamiltonian Dynamics approach. In a frame where $\mathbf{P}_\perp = \mathbf{0}$, one has

$$n(k_\perp) = \frac{N_c}{P_{q\bar{q}}(2\pi)^3 [P^+]^2} \int_0^{2\pi} d\theta_{\hat{k}_\perp} \int_0^1 \frac{d\xi}{\xi(1-\xi)} M_0^2 |\Phi(\xi, \mathbf{k}_\perp; m_{R_i})|^2 \quad (4)$$

where N_c is the number of colors, $k_\perp = |\mathbf{k}_\perp|$ and $\Phi(\xi, \mathbf{k}_\perp; m_{R_i})$ is the valence wave function associated to a given BS amplitude, see, e.g., Huang and Karmanov⁴⁾ and Frederico et al³⁾. In Eq. (4), the probability $P_{q\bar{q}}$ of the valence component reads

$$P_{q\bar{q}} = \frac{N_c}{(2\pi)^3 [P^+]^2} \int_0^1 \frac{d\xi}{\xi(1-\xi)} \int d\mathbf{k}_\perp M_0^2 |\Phi(\xi, \mathbf{k}_\perp; m_{R_i})|^2 \quad (5)$$

Finally, $n(k_\perp)$ is normalized as: $\int k_\perp dk_\perp n(k_\perp) = 1$.

In spite of the simple form assumed for Λ_{VM} , one can nicely fit the constituent transverse momentum distributions obtained within 3-D approaches, that i) retain only the valence component of the VM's and ii) are able to yield a reasonable description of the spectrum. In this work we have extracted the parameters m_{R_i} in Eq. (4) by fitting $n(k_\perp)$ to the corresponding quantity obtained from i) a Harmonic Oscillator model (see, e.g. Figs. 1 and 2), ii) the Godfrey-Isgur model⁵⁾ and iii) an adapted version of the model by Salcedo et al⁶⁾ (ITA model).

2 The Mandelstam formula for em decay constant

In order to evaluate the em decays constants, f_V , we adopted a Mandelstam-like formula⁷⁾ (see also de Melo et al.⁸⁾). The starting point is the *macroscopic* definition of f_V , through the transition matrix element of the em current for a given neutral VM, viz

$$\langle 0 | J^\mu(0) | P, \lambda \rangle = i\sqrt{2} f_V \epsilon_\lambda^\mu \quad (6)$$

The decay constant f_V is related to the em decay width as follows

$$\Gamma_{e^+e^-} = \frac{8\pi\alpha^2}{3} \frac{|f_V|^2}{M^3} \quad (7)$$

In our model, the transition matrix element in Eq. (6) can be approximated *microscopically à la* Mandelstam through

$$\langle 0 | J^\mu(0) | P, \lambda \rangle = \mathcal{F}_{VM} \frac{N_c \mathcal{N}}{(2\pi)^4} \int d^4k \frac{\Lambda_{VM}(k, k-P, m_1, m_2)}{(k^2 - m_1^2 + i\epsilon) [(P-k)^2 - m_2^2 + i\epsilon]} \times$$

Table 1: Preliminary VM em decay widths within the Harmonic Oscillator model. Adopted quark masses: $m_u = 0.310$ GeV, $m_s = 0.460$ GeV, $m_c = 1.749$ GeV, $m_b = 5.068$ GeV,

VM	m_{VM} (MeV)	$P_{q\bar{q}}$	$\Gamma_{e^+e^-}^{th}$ (keV)	$\Gamma_{e^+e^-}^{exp}$ (keV)
ρ	775.5 ± 0.4	0.884	10.328	7.02 ± 0.11
ϕ	1019.460 ± 0.019	0.961	1.582	1.32 ± 0.06
J/ψ	3096.916 ± 0.011	0.787	1.572	5.55 ± 0.14

Table 2: Preliminary VM em decay widths within the Godfrey-Isgur ⁵⁾ model, $m_u = 0.220$ GeV, $m_s = 0.419$ GeV, $m_c = 1.628$ GeV, $m_b = 4.977$ GeV.

VM	m_{VM} (MeV)	$P_{q\bar{q}}$	$\Gamma_{e^+e^-}^{th}$ (KeV)	$\Gamma_{e^+e^-}^{exp}$ (keV)
ρ	775.5 ± 0.4	0.411	18.098	7.02 ± 0.11
ϕ	1019.460 ± 0.019	0.906	3.733	1.32 ± 0.06
J/ψ	3096.916 ± 0.011	0.908	5.911	5.55 ± 0.14

$$\text{Tr}[\epsilon_\lambda(P) \cdot V(P) (\not{k} - \not{P} + m_2) \gamma^\mu (\not{k} + m_1)] \quad (8)$$

where

$$\mathcal{F}_\rho = \frac{(Q_u - Q_d)}{\sqrt{2}} \quad \mathcal{F}_\phi = Q_s \quad \mathcal{F}_{J/\Psi} = Q_c$$

with Q_i the quark charge. In Tabs. 1, 2, 3, the preliminary results for both valence probability, $P_{q\bar{q}}$, and em decay widths, $\Gamma_{e^+e^-}$ are shown. Even if a more refined evaluations are in progress, some comments are in order: i) for the Harmonic Oscillator model the light meson decay widths can be reasonably well described (in the light sector the confining interaction is quite relevant), while the J/ψ one is largely underestimated; ii) for the Godfrey-Isgur model ⁵⁾, the heavy sector is well reproduced, while the light sector is overestimated, and this appears correlated to the poor estimate of the valence probability (work in progress suggests that an Ansatz for the BS amplitude with a more rich structure substantially improves the comparison); iii) for the adapted version of the ITA model ⁶⁾ the same pattern of the Harmonic Oscillator case has been found, even if more dynamical contents are present in this model.

Table 3: Preliminary VM em decay widths within an adapted version of the ITA model. ⁶⁾. Adopted quark masses: $m_u = 0.334$ GeV, $m_s = 0.460$ GeV, $m_c = 1.791$ GeV, $m_b = 4.679$ GeV.

VM	m_{VM} (MeV)	$P_{q\bar{q}}$	$\Gamma_{e^+e^-}^{th}$ (keV)	$\Gamma_{e^+e^-}^{exp}$ (keV)
ρ	775.5 ± 0.4	0.913	7.548	7.02 ± 0.11
ϕ	1019.460 ± 0.019	0.995	1.294	1.32 ± 0.06
J/ψ	3096.916 ± 0.011	0.726	1.250	5.55 ± 0.14

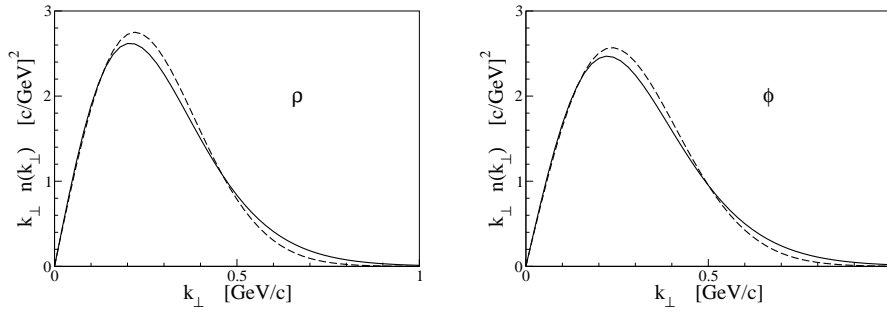


Figure 1: Transverse momentum distributions for a constituent inside ρ and ϕ vs the quark transverse momentum. Dashed line: Harmonic Oscillator model. Solid line: fit by using the analytic Ansatz in Eq. (3)

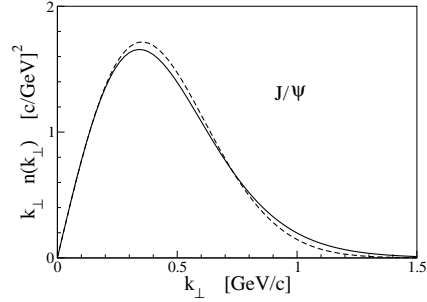


Figure 2: The same as in Fig. 1, but for J/ψ .

3 Conclusions

In this contribution we have presented the main ingredients of our model for evaluating both the em decay width of the ground states of VM's and the probability of the valence component of the state. In our fully covariant model, a simple, analytic Ansatz for the BS amplitude is proposed, and the three parameters, m_{Ri} , for each neutral VM, are determined through a fitting procedure, based on the transverse momentum distribution of a constituent inside a given VM, obtained within a Light-Front Hamiltonian Dynamics framework. From this first comparisons between our results and the experimental data, one could argue that two different regimes occur in the light (ρ, ϕ) and in the heavy sector (J/ψ). From the theoretical side, indeed, the Harmonic Oscillator and the adapted ITA⁶⁾ models seem to better reproduce the light mesons (cf Tab. 1, 2) while for the heavy sector the Godfrey-Isgur model⁵⁾ seems to work better (cf. Tab. 3).

The work in progress will substantially improve the present calculations, in two respect: both introducing a more refined Ansatz for the BS amplitude and extending our investigation to the em decay of the Υ .

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